Finding the Greatest Common Factor of Polynomials

In a multiplication problem, the numbers multiplied together are called **factors**. The answer to a multiplication problem is called the **product**.

In the multiplication problem $5 \times 4 = 20$, 5 and 4 are **factors** and 20 is the **product**.

If we reverse the problem, $20 = 5 \times 4$, we say we have **factored** 20 into 5×4.

In this worksheet we will factor polynomials.

In the multiplication problem $2x(x+4) = 2x^2 + 8x$, $2x$ and $x+4$ are the **factors** and $2x^2 + 8x$ is the **product**.

If we reverse the problem, $2x^2 + 8x = 2x(x+4)$, we say we have **factored** $2x^2 + 8x$ into $2x$ and $x+4$.

Name the factors and the product in each problem.

1. $5(x-7) = 5x - 35$ factors: _____ product: _____
2. $3x(x+9) = 3x^2 + 27x$ factors: _____ product: _____
3. $-10x(x-6) = -10x^2 + 60x$ factors: _____ product: _____
4. $4xy^2(3x+8y) = 12x^2y^2 + 32xy^3$ factors: _____ product: _____

The first step in factoring polynomials is to factor out the **greatest common factor (GCF)**. This is the **largest integer** and **highest degree of each variable** that will divide evenly into each term of the polynomial.

Factoring is the reverse of multiplying!

- In the polynomial $5x - 35$, 5 is the largest integer that will divide 5x and 35, and we cannot factor out any variable because the second term, 35, does not have a variable part.

  To factor $5x - 35$ we write: $5x - 35 = 5(x-7)$.

- In the polynomial $3x^2 + 27x$, 3 is the largest integer that will divide $3x^2$ and $27x$. We can factor out x because each term has at least one factor of x (look for the term with the lowest degree of each variable).

  To factor $3x^2 + 27x$ we write: $3x^2 + 27x = 3x(x+9)$.

- In the polynomial $12x^2y^2 + 32xy^3$, 4 is the largest integer that will divide $12x^2y^2$ and $32xy^3$. We can factor out x and y^2 because each term has at least one factor of x and two factors of y.

  To factor $12x^2y^2 + 32xy^3$ we write: $12x^2y^2 + 32xy^3 = 4xy^2(3x+8y)$.

Finding the Greatest Common Factor of Polynomials

Find the largest integer that will divide all the terms.

5. $9x$ and 45

6. $7x^2$ and $21x$

7. $18x^6$ and $12x^3$

8. $15x^3, 25x^2,$ and $55x$

Find the largest degree of x that can be factored out of all the terms.

9. $9x$ and 45

10. $7x^2$ and $21x$

11. $18x^6$ and $12x^3$

12. $15x^3, 25x^2,$ and $55x$

Factor the polynomials.

13. $9x + 45 =$

14. $7x^2 - 21x =$

15. $18x^6 + 12x^3 =$

16. $15x^3 - 25x^2 + 55x =$

To factor polynomials, find the greatest common factor (GCF) of the coefficients and factor it out- divide each term by the GCF. Then find the greatest common factor (GCF) of the variables by finding the lowest power of each variable that will divide all terms and factor it out- divide each term by GCF. Move the GCF to the outside and write in parenthesis what is remaining, after you factor out the GCF.

Factor each of the following polynomials.

17. $6x^2 - 24x$

18. $14x^2 - 35x$

19. $5x^2 + x$

20. $20x^2 + 44x$

21. $17x^2 + 51x$

22. $36x^3 + 63x^2 - 27x$

23. $3x^4y^2 + 15x^3y^3$

24. $20y^4 - 15y^3 + 30y^2$

25. $9x^7y^5 - 3x^2y^6$

If the leading coefficient is negative, always factor out the negative!

26. $-2m^4 + 14m^2 - 6m$

27. $-5x^2y + 35xy$

28. $-x^2 + 5x - 6$

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Name the factors and the product in each problem.

1. $5(x-7) = 5x - 35$ factors: 5, (x-7) product: 5x-35
2. $3x(x+9) = 3x^2 + 27x$ factors: 3x, (x+9) product: 3x^2+27x
3. $-10x(x-6) = -10x^2 + 60x$ factors: -10x, (x-6) product: -10x^2+60x
4. $4xy^2(3x+8y) = 12x^2y^2 + 32xy^3$ factors: 4xy^2, (3x+8y) product: 12x^2y^2+32xy^3

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09

6. $7x^2$ and $21x$
7

7. $18x^6$ and $12x^3$
6

8. $15x^3, 25x^2,$ and $55x$
5

Find the largest degree of x that can be factored out of all the terms.

9. $9x$ and 45
0

10. $7x^2$ and $21x$
x

11. $18x^6$ and $12x^3$
x^3

12. $15x^3, 25x^2,$ and $55x$
x

Factor the polynomials.

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9(x+5)

14. $7x^2 - 21x =$
7x(x-3)

15. $18x^6 + 12x^3 =$
6x^3(3x^3+2)

16. $15x^3 - 25x^2 + 55x =$
5x(3x^2-5x+11)

To factor polynomials, find the greatest common factor (GCF) of the coefficients and factor it out- divide each term by the GCF. Then find the greatest common factor (GCF) of the variables by finding the lowest power of each variable that will divide all terms and factor it out- divide each term by GCF. Move the GCF to the outside and write in parenthesis what is remaining, after you factor out the GCF.

Factor each of the following polynomials.

17. $6x^2 - 24x$
6x(x-4)

18. $14x^2 - 35x$
7x(2x-5)

19. $5x^2 + x$
x(5x+1)

20. $20x^2 + 44x$
4x(5x+11)

21. $17x^2 + 51x$
17x(x+3)

22. $36x^3 + 63x^2 - 27x$
9x(4x^2+7x-3)

23. $3x^4y^2 + 15x^3y^3$
3x^3y^2(x+5y)

24. $20y^4 - 15y^3 + 30y^2$
5y^2(4y^2-3y+6)

25. $9x^7y^5 - 3x^2y^6$
3x^2y^5(3x^5-y)

If the leading coefficient is negative, always factor out the negative!

26. $-2m^4 + 14m^2 - 6m$
-2m(m^3-7m+3)

27. $-5x^2y + 35xy$
-5xy(x-7)

28. $-x^2 + 5x - 6$
-(x^2-5x+6)