Lesson 2

Understand Rational and Irrational Numbers

Name: _____

Prerequisite: How can you express fractions as repeating or terminating decimals?

Study the example problem showing how to use division to express fractions as repeating decimals. Then solve problems 1–7.

Example

Erika uses division to write $\frac{1}{3}$ and $\frac{2}{3}$ as decimals. First she estimates that because $\frac{1}{3}$ is between $\frac{1}{4}$ and $\frac{1}{2}$, it will be between 0.25 and 0.5. Likewise, because $\frac{2}{3}$ is between $\frac{1}{2}$ and $\frac{3}{4}$, it will be between 0.5 and 0.75. Then she divides as shown at the right.

$\frac{1}{3} = 0.333\ldots$, or $0.\overline{3}$ $\quad \frac{2}{3} = 0.666\ldots$, or $0.\overline{6}$

$$\begin{array}{r|l} & 0.333 \\ 3 & 1.000 \\ -9 & \\ \hline 10 & \\ -9 & \\ \hline 10 & \\ -9 & \\ \hline 1 & \end{array} \qquad \begin{array}{r|l} & 0.666 \\ 3 & 2.000 \\ -18 & \\ \hline 20 & \\ -18 & \\ \hline 20 & \\ -18 & \\ \hline 2 & \end{array}$$

B 1 Erika says that no matter how many decimal places she divides to when she divides 1 by 3, the digit 3 in the quotient will just keep repeating. Is she correct? Explain.

Yes; Possible explanation: She keeps subtracting 9 from 10 and dividing 10 by 3, so the digit 3 keeps repeating.

B 2 Is the decimal for $\frac{4}{3}$ a *repeating decimal*? Explain.

Yes; Possible explanation: When I divide 4 by 3, the 3 keeps repeating in the quotient.

M 3 How could Erika have used the decimal that she wrote for $\frac{1}{3}$ to find the decimal for $\frac{2}{3}$?

Possible answer: The decimal for $\frac{2}{3}$ is double the decimal for $\frac{1}{3}$, which makes sense because $\frac{2}{3}$ is twice as much as $\frac{1}{3}$.

Vocabulary

repeating decimals decimals that repeat the same digit or sequence of digits forever. A repeating decimal can be written with a bar over the repeating digits.

0.333 . . . and 0.1666 . . . are repeating decimals.

Solve.

B 4 Write $\frac{1}{8}$ as a decimal. Explain why this decimal is called a *terminating decimal*.

0.125; Possible explanation: The decimal ends, or terminates.

M 5 Tell whether each statement below is true or false. If it is false, write an example that proves the statement is false.

a. All fractions can be written as terminating decimals.

false; Possible explanation: $\frac{4}{9}$ can be written as a repeating decimal but not a terminating decimal.

b. If a fraction can be written as a repeating decimal, only one digit can repeat over and over, without end.

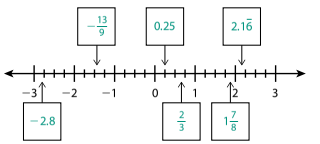
false; Possible explanation: $\frac{1}{11}$ can be written as .09090909 . . . , which has two repeating digits.

M 6 Raj is playing a game and has the cards below. He needs to find pairs of cards that have the same value. Which two pairs of cards express the same value?

$\frac{3}{8}$ and 0.375, $\frac{7}{9}$ and $0.\overline{7}$

C 7 Write each number in the appropriate box to show its location along the number line.

-2.8	$\frac{2}{3}$	$2.1\overline{6}$	$1\frac{7}{8}$	0.25	$-\frac{13}{9}$



Vocabulary

terminating decimals decimals that end, or end in repeated zeros.

0.5; 4.08; 0.300

Key

B Basic **M** Medium **C** Challenge

Lesson 2

Name: _____

Approximate Irrational Numbers

Study the example problem showing how to approximate the value of an irrational number. Then solve problems 1–8.

Example

Approximate the value of $\sqrt{6}$ to the nearest hundredth.

The value of $\sqrt{6}$ is between $\sqrt{4}$ and $\sqrt{9}$. So, $\sqrt{6}$ is between 2 and 3, but closer to 2 than to 3.

Find the squares of tenths that are closer to 2 than to 3 to find which two tenths $\sqrt{6}$ is between.

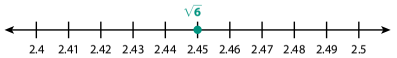
$2.3^2 = 5.29$ $2.4^2 = 5.76$ $2.5^2 = 6.25$

Because 6 is almost exactly halfway between 5.76 and 6.25, $\sqrt{6}$ must be almost exactly halfway between 2.4 and 2.5. Now you can find which two hundredths $\sqrt{6}$ is between.

$2.44^2 = 5.9536$ and $2.45^2 = 6.0025$

$\sqrt{6}$ is between 2.44 and 2.45, but it is closer to 2.45.

B 1 Plot $\sqrt{6}$ on the number line showing its approximate location to the nearest hundredth.



B 2 Check your answer by finding $\sqrt{6}$ using a calculator. What is the result on your screen?

2.449489743

M 3 Find $3\sqrt{10}$ to the nearest hundredth. Explain how you found your answer.

9.48 or 9.49; Possible explanation: I found that $\sqrt{10}$ is about 3.162, and 3 times this value is 9.486, which rounds to 9.49.

Vocabulary

irrational number a number that cannot be expressed as a ratio of two integers. The decimal expansion of an irrational number never repeats or terminates.

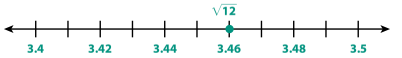
$\sqrt{3}$ is an irrational number.

Solve.

B 4 Explain how a rational number and an irrational number are different.

Possible explanation: A rational number can be written as a ratio of two integers while an irrational number cannot.

M 5 Plot $\sqrt{12}$ on the number line showing its location to the nearest hundredth.



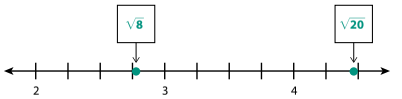
M 6 Is 1.75 a reasonable approximation of the value of $\sqrt{8}$? Explain your reasoning.

No; Possible explanation: $\sqrt{8}$ is between $\sqrt{4}$ and $\sqrt{9}$, or between 2 and 3. Because 1.75 is less than 2, it is not a reasonable approximation.

M 7 On a number line, is $\sqrt{20}$ closer to 4.4 or 4.5? Explain your reasoning.

Closer to 4.5; Possible explanation: $4.4^2 = 19.36$ and $4.5^2 = 20.25$. 20.25 is closer to 20 than 19.36 is to 20.

C 8 Look at the two points on the number line. Each number plotted is the square root of a whole number that is not a perfect square. Write the appropriate square root in each box. Explain how you found your answers.



Possible explanation: I knew that the first square root was between $\sqrt{4}$ and $\sqrt{9}$ and just a little more than 2.75. I squared 2.75 and found that it was 7.5625. The closest whole number is 8, so I labeled that point $\sqrt{8}$. I found my other answer the same way.

Vocabulary

rational number a number that can be expressed as a ratio $\frac{a}{b}$ where a and b are integers and $b \neq 0$. Rational numbers also include integers, repeating decimals, and terminating decimals.

$2.5 = \frac{25}{10}$

$0.8333\ldots = \frac{5}{6}$

Lesson 2

Name: _____

Reason and Write

Study the example problem. Underline two parts that you think make it a particularly good answer and a helpful example.

Answers will vary. Note whether students incorporate the features they chose in their answer on the next page.

Example

Plot $-3 + \sqrt{17}$ on a number line showing its approximate location to the nearest hundredth. Explain how you approximated and plotted the value.

Show your work. Use decimal approximations, a number line, and words to explain your answer.

Possible answer:

First, I need to approximate the value of $\sqrt{17}$, the irrational number.

$\sqrt{17}$ is between 4 and 5 because $4^2 = 16$ and $5^2 = 25$. It is closer to 4 than to 5.

Since $4.1^2 = 16.81$ and $4.2^2 = 17.64$, $\sqrt{17}$ is between 4.1 and 4.2. It is closer to 4.1.

Now $4.12^2 \approx 16.97$ and $4.13^2 \approx 17.06$. So, $\sqrt{17} \approx 4.12$ since 4.12^2 is closest to 17.

Now I add 4.12 to −3 to get the value I need to plot on the number line.

$$-3 + 4.12 = 1.12$$

Where does the example . . .
- show how to approximate the number?
- use a decimal approximation?
- include a number line?
- use words to explain?

Solve the problem. Use what you learned from the model.

Plot $8 - \sqrt{28}$ on a number line showing its approximate location to the nearest hundredth. Explain how you approximated and plotted the value.

Show your work. Use decimal approximations, a number line, and words to explain your answer.

Possible answer:

First, I need to approximate the value of $\sqrt{28}$, the irrational number.

$\sqrt{28}$ is between 5 and 6 because $5^2 = 25$ and $6^2 = 36$. It is closer to 5 than to 6.

Since $5.2^2 = 27.04$ and $5.3^2 = 28.09$, $\sqrt{28}$ is between 5.2 and 5.3. It is closer to 5.3.

Now $5.29^2 \approx 27.98$ and $5.3^2 = 28.09$. So, $\sqrt{28} \approx 5.29$ since 5.29^2 is closest to 28.

Now I subtract 5.29 from 8 to get the value I need to plot on the number line.

$$8 - 5.29 = 2.71$$

Did you . . .
- show how to approximate the number?
- include a decimal approximation?
- include a number line?
- use words to explain?