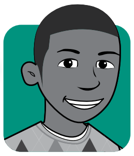
# *Understand* Rational and Irrational Numbers

Name: ____________________

## Prerequisite: How can you express fractions as repeating or terminating decimals?



**Study the example problem showing how to use division to express fractions as repeating decimals. Then solve problems 1–7.**

### Example

Erika uses division to write $\frac{1}{3}$ and $\frac{2}{3}$ as decimals. First she estimates that because $\frac{1}{3}$ is between $\frac{1}{4}$ and $\frac{1}{2}$, it will be between 0.25 and 0.5. Likewise, because $\frac{2}{3}$ is between $\frac{1}{2}$ and $\frac{3}{4}$, it will be between 0.5 and 0.75. Then she divides as shown at the right.

$\frac{1}{3} = 0.333\ldots$, or $0.\overline{3}$ $\qquad$ $\frac{2}{3} = 0.666\ldots$, or $0.\overline{6}$

$$\begin{array}{r} 0.333 \\ 3\overline{)1.000} \\ -9\phantom{00} \\ \hline 10\phantom{0} \\ -9\phantom{0} \\ \hline 10 \\ -9 \\ \hline 1 \end{array} \qquad \begin{array}{r} 0.666 \\ 3\overline{)2.000} \\ -1\,8\phantom{00} \\ \hline 20\phantom{0} \\ -18\phantom{0} \\ \hline 20 \\ -18 \\ \hline 2 \end{array}$$

**1** Erika says that no matter how many decimal places she divides to when she divides 1 by 3, the digit 3 in the quotient will just keep repeating. Is she correct? Explain.

**2** Is the decimal for $\frac{4}{3}$ a *repeating decimal*? Explain.

**3** How could Erika have used the decimal that she wrote for $\frac{1}{3}$ to find the decimal for $\frac{2}{3}$?

### Vocabulary

**repeating decimals** decimals that repeat the same digit or sequence of digits forever. A repeating decimal can be written with a bar over the repeating digits.

0.333 . . . and 0.1666 . . . are repeating decimals.

**Solve.**

**4** Write $\frac{1}{8}$ as a decimal. Explain why this decimal is called a *terminating decimal*.

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**5** Tell whether each statement below is true or false. If it is false, write an example that proves the statement is false.

**a.** All fractions can be written as terminating decimals.

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**b.** If a fraction can be written as a repeating decimal, only one digit can repeat over and over, without end.

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**6** Raj is playing a game and has the cards below. He needs to find pairs of cards that have the same value. Which two pairs of cards express the same value?

| | | | | | | |
|---|---|---|---|---|---|---|
| $\frac{7}{9}$ | $\frac{3}{8}$ | $\frac{3}{5}$ | 0.375 | $0.\overline{9}$ | $0.\overline{7}$ | 0.675 |

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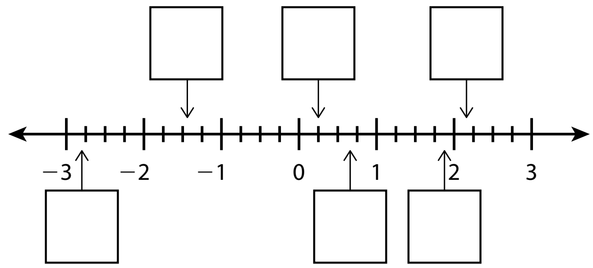
**7** Write each number in the appropriate box to show its location along the number line.

| | | | | | |
|---|---|---|---|---|---|
| −2.8 | $\frac{2}{3}$ | $2.1\overline{6}$ | $1\frac{7}{8}$ | 0.25 | $-\frac{13}{9}$ |




## Vocabulary

**terminating decimals**
decimals that end, or end in repeated zeros.

0.5; 4.08; 0.300

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## Approximate Irrational Numbers

**Study the example problem showing how to approximate the value of an irrational number. Then solve problems 1–8.**

**Example**

Approximate the value of $\sqrt{6}$ to the nearest hundredth.

The value of $\sqrt{6}$ is between $\sqrt{4}$ and $\sqrt{9}$. So, $\sqrt{6}$ is between 2 and 3, but closer to 2 than to 3.

Find the squares of tenths that are closer to 2 than to 3 to find which two tenths $\sqrt{6}$ is between.

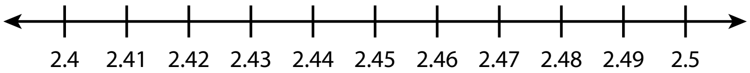
$$2.3^2 = 5.29 \qquad 2.4^2 = 5.76 \qquad 2.5^2 = 6.25$$

Because 6 is almost exactly halfway between 5.76 and 6.25, $\sqrt{6}$ must be almost exactly halfway between 2.4 and 2.5. Now you can find which two hundredths $\sqrt{6}$ is between.

$$2.44^2 = 5.9536 \text{ and } 2.45^2 = 6.0025$$

$\sqrt{6}$ is between 2.44 and 2.45, but it is closer to 2.45.

**1** Plot $\sqrt{6}$ on the number line showing its approximate location to the nearest hundredth.




**2** Check your answer by finding $\sqrt{6}$ using a calculator. What is the result on your screen?

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**3** Find $3\sqrt{10}$ to the nearest hundredth. Explain how you found your answer.

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**Vocabulary**

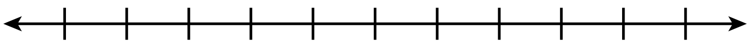
**irrational number**
a number that cannot be expressed as a ratio of two integers. The decimal expansion of an irrational number never repeats or terminates.

$\sqrt{3}$ is an irrational number.

**Solve.**

**4** Explain how a rational number and an irrational number are different.

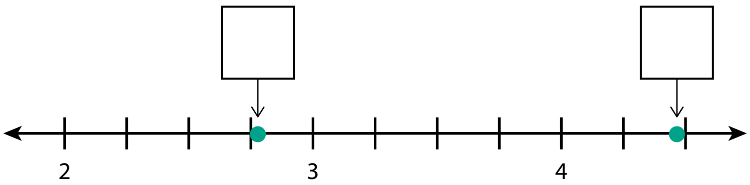
**5** Plot $\sqrt{12}$ on the number line showing its location to the nearest hundredth.



**6** Is 1.75 a reasonable approximation of the value of $\sqrt{8}$? Explain your reasoning.

**7** On a number line, is $\sqrt{20}$ closer to 4.4 or 4.5? Explain your reasoning.

**8** Look at the two points on the number line. Each number plotted is the square root of a whole number that is not a perfect square. Write the appropriate square root in each box. Explain how you found your answers.




## Vocabulary

**rational number** a number that can be expressed as a ratio $\frac{a}{b}$ where $a$ and $b$ are integers and $b \neq 0$. Rational numbers also include integers, repeating decimals, and terminating decimals.

$2.5 = \frac{25}{10}$

$0.8333 \ldots = \frac{5}{6}$