

## Basic Properties and Facts

### Arithmetic Operations

$$\begin{aligned}
 ab + ac &= a(b + c) & a\left(\frac{b}{c}\right) &= \frac{ab}{c} \\
 \frac{\left(\frac{a}{b}\right)}{c} &= \frac{a}{bc} & \frac{a}{\left(\frac{b}{c}\right)} &= \frac{ac}{b} \\
 \frac{a}{b} + \frac{c}{d} &= \frac{ad + bc}{bd} & \frac{a}{b} - \frac{c}{d} &= \frac{ad - bc}{bd} \\
 \frac{a - b}{c - d} &= \frac{b - a}{d - c} & \frac{a + b}{c} &= \frac{a}{c} + \frac{b}{c} \\
 \frac{ab + ac}{a} &= b + c, \ a \neq 0 & \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} &= \frac{ad}{bc}
 \end{aligned}$$

### Exponent Properties

$$\begin{aligned}
 a^n a^m &= a^{n+m} & (ab)^n &= a^n b^n \\
 (a^n)^m &= a^{nm} & a^0 &= 1, \ a \neq 0 \\
 \frac{a^n}{a^m} &= a^{n-m} = \frac{1}{a^{m-n}} & \left(\frac{a}{b}\right)^n &= \frac{a^n}{b^n} \\
 a^{\frac{n}{m}} &= \left(a^{\frac{1}{m}}\right)^n = (a^n)^{\frac{1}{m}} & \frac{1}{a^{-n}} &= a^n \\
 \left(\frac{a}{b}\right)^{-n} &= \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n} & a^{-n} &= \frac{1}{a^n}
 \end{aligned}$$

### Properties of Radicals

$$\begin{aligned}
 \sqrt[n]{a} &= a^{\frac{1}{n}} & \sqrt[n]{ab} &= \sqrt[n]{a} \sqrt[n]{b} \\
 \sqrt[m]{\sqrt[n]{a}} &= \sqrt[nm]{a} & \sqrt[n]{\frac{a}{b}} &= \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \\
 \sqrt[n]{a^n} &= a \text{ if } n \text{ is odd} \\
 \sqrt[n]{a^n} &= |a| \text{ if } n \text{ is even}
 \end{aligned}$$

### Properties of Inequalities

If  $a < b$  then  $a + c < b + c$  and  $a - c < b - c$

If  $a < b$  and  $c > 0$  then  $ac < bc$  and  $\frac{a}{c} < \frac{b}{c}$

If  $a < b$  and  $c < 0$  then  $ac > bc$  and  $\frac{a}{c} > \frac{b}{c}$

### Properties of Absolute Value

$$\begin{aligned}
 |a| &= \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases} \\
 |a| &\geq 0 & |-a| &= |a| \\
 |ab| &= |a| |b| & \left|\frac{a}{b}\right| &= \frac{|a|}{|b|} \\
 |a + b| &\leq |a| + |b| & \text{Triangle Inequality}
 \end{aligned}$$

### Distance Formula

If  $P_1 = (x_1, y_1)$  and  $P_2 = (x_2, y_2)$  are two points the distance between them is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

### Complex Numbers

$$i = \sqrt{-1} \quad i^2 = -1 \quad \sqrt{-a} = i\sqrt{a}, \ a \geq 0$$

$$(a + bi) + (c + di) = a + c + (b + d)i$$

$$(a + bi) - (c + di) = a - c + (b - d)i$$

$$(a + bi)(c + di) = ac - bd + (ad + bc)i$$

$$(a + bi)(a - bi) = a^2 + b^2$$

$$|a + bi| = \sqrt{a^2 + b^2} \quad \text{Complex Modulus}$$

$$\overline{(a + bi)} = a - bi \quad \text{Complex Conjugate}$$

$$\overline{(a + bi)}(a + bi) = |a + bi|^2$$

**Logarithms and Log Properties****Definition**

$y = \log_b(x)$  is equivalent to  $x = b^y$

**Example**

$\log_5(125) = 3$  because  $5^3 = 125$

**Special Logarithms**

$\ln(x) = \log_e(x)$  natural log

$\log(x) = \log_{10}(x)$  common log

where  $e = 2.718281828 \dots$

**Logarithm Properties**

$$\log_b(b) = 1 \qquad \log_b(1) = 0$$

$$\log_b(b^x) = x \qquad b^{\log_b(x)} = x$$

$$\log_b(x^r) = r \log_b(x)$$

$$\log_b(xy) = \log_b(x) + \log_b(y)$$

$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$$

The domain of  $\log_b(x)$  is  $x > 0$

**Factoring and Solving****Factoring Formulas**

$$x^2 - a^2 = (x + a)(x - a)$$

$$x^2 + 2ax + a^2 = (x + a)^2$$

$$x^2 - 2ax + a^2 = (x - a)^2$$

$$x^2 + (a + b)x + ab = (x + a)(x + b)$$

$$x^3 + 3ax^2 + 3a^2x + a^3 = (x + a)^3$$

$$x^3 - 3ax^2 + 3a^2x - a^3 = (x - a)^3$$

$$x^3 + a^3 = (x + a)(x^2 - ax + a^2)$$

$$x^3 - a^3 = (x - a)(x^2 + ax + a^2)$$

$$x^{2n} - a^{2n} = (x^n - a^n)(x^n + a^n)$$

If  $n$  is odd then,

$$x^n - a^n = (x - a)(x^{n-1} + ax^{n-2} + \dots + a^{n-1})$$

$$x^n + a^n = (x + a)(x^{n-1} - ax^{n-2} + a^2x^{n-3} - \dots + a^{n-1})$$

**Quadratic Formula**

Solve  $ax^2 + bx + c = 0$ ,  $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If  $b^2 - 4ac > 0$  – Two real unequal solns.

If  $b^2 - 4ac = 0$  – Repeated real solution.

If  $b^2 - 4ac < 0$  – Two complex solutions.

**Square Root Property**

If  $x^2 = p$  then  $x = \pm\sqrt{p}$

**Absolute Value Equations/Inequalities**

If  $b$  is a positive number

$$|p| = b \Rightarrow p = -b \text{ or } p = b$$

$$|p| < b \Rightarrow -b < p < b$$

$$|p| > b \Rightarrow p < -b \text{ or } p > b$$

**Completing the Square**

Solve  $2x^2 - 6x - 10 = 0$

(1) Divide by the coefficient of the  $x^2$

$$x^2 - 3x - 5 = 0$$

(2) Move the constant to the other side.

$$x^2 - 3x = 5$$

(3) Take half the coefficient of  $x$ , square it and add it to both sides

$$x^2 - 3x + \left(-\frac{3}{2}\right)^2 = 5 + \left(-\frac{3}{2}\right)^2 = 5 + \frac{9}{4} = \frac{29}{4}$$

(4) Factor the left side

$$\left(x - \frac{3}{2}\right)^2 = \frac{29}{4}$$

(5) Use Square Root Property

$$x - \frac{3}{2} = \pm\sqrt{\frac{29}{4}} = \pm\frac{\sqrt{29}}{2}$$

(6) Solve for  $x$

$$x = \frac{3}{2} \pm \frac{\sqrt{29}}{2}$$

**Functions and Graphs****Constant Function**

$$y = a \quad \text{or} \quad f(x) = a$$

Graph is a horizontal line passing through the point  $(0, a)$ .

**Line/Linear Function**

$$y = mx + b \quad \text{or} \quad f(x) = mx + b$$

Graph is a line with point  $(0, b)$  and slope  $m$ .

Slope

Slope of the line containing the two points  $(x_1, y_1)$  and  $(x_2, y_2)$  is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

Slope – intercept form

The equation of the line with slope  $m$  and  $y$ -intercept  $(0, b)$  is

$$y = mx + b$$

Point – Slope form

The equation of the line with slope  $m$  and passing through the point  $(x_1, y_1)$  is

$$y = y_1 + m(x - x_1)$$

**Parabola/Quadratic Function**

$$y = a(x - h)^2 + k \quad f(x) = a(x - h)^2 + k$$

The graph is a parabola that opens up if  $a > 0$  or down if  $a < 0$  and has a vertex at  $(h, k)$ .

**Parabola/Quadratic Function**

$$y = ax^2 + bx + c \quad f(x) = ax^2 + bx + c$$

The graph is a parabola that opens up if  $a > 0$  or down if  $a < 0$  and has a vertex at

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right).$$

**Parabola/Quadratic Function**

$$x = ay^2 + by + c \quad g(y) = ay^2 + by + c$$

The graph is a parabola that opens right if  $a > 0$  or left if  $a < 0$  and has a vertex at

$$\left(g\left(-\frac{b}{2a}\right), -\frac{b}{2a}\right).$$

**Circle**

$$(x - h)^2 + (y - k)^2 = r^2$$

Graph is a circle with radius  $r$  and center  $(h, k)$ .

**Ellipse**

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

Graph is an ellipse with center  $(h, k)$  with vertices  $a$  units right/left from the center and vertices  $b$  units up/down from the center.

**Hyperbola**

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

Graph is a hyperbola that opens left and right, has a center at  $(h, k)$ , vertices  $a$  units left/right of center and asymptotes that pass through center with slope  $\pm \frac{b}{a}$ .

**Hyperbola**

$$\frac{(y - k)^2}{b^2} - \frac{(x - h)^2}{a^2} = 1$$

Graph is a hyperbola that opens up and down, has a center at  $(h, k)$ , vertices  $b$  units up/down from the center and asymptotes that pass through center with slope  $\pm \frac{b}{a}$ .

**Common Algebraic Errors**

Error	Reason/Correct/Justification/Example
$\frac{2}{0} \neq 0$ and $\frac{2}{0} \neq 2$	Division by zero is undefined!
$-3^2 \neq 9$	$-3^2 = -9$ , $(-3)^2 = 9$ Watch parenthesis!
$(x^2)^3 \neq x^5$	$(x^2)^3 = x^2 x^2 x^2 = x^6$
$\frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$	$\frac{1}{2} = \frac{1}{1+1} \neq \frac{1}{1} + \frac{1}{1} = 2$
$\frac{1}{x^2+x^3} \neq x^{-2} + x^{-3}$	A more complex version of the previous error.
$\frac{a+bx}{a} \neq 1+bx$	$\frac{a+bx}{a} = \frac{a}{a} + \frac{bx}{a} = 1 + \frac{bx}{a}$ Beware of incorrect canceling!
$-a(x-1) \neq -ax-a$	$-a(x-1) = -ax+a$ Make sure you distribute the "-".
$(x+a)^2 \neq x^2+a^2$	$(x+a)^2 = (x+a)(x+a) = x^2+2ax+a^2$
$\sqrt{x^2+a^2} \neq x+a$	$5 = \sqrt{25} = \sqrt{3^2+4^2} \neq \sqrt{3^2} + \sqrt{4^2} = 3+4=7$
$\sqrt{x+a} \neq \sqrt{x} + \sqrt{a}$	See previous error.
$(x+a)^n \neq x^n+a^n$ and $\sqrt[n]{x+a} \neq \sqrt[n]{x} + \sqrt[n]{a}$	More general versions of previous three errors.
$2(x+1)^2 \neq (2x+2)^2$	$2(x+1)^2 = 2(x^2+2x+1) = 2x^2+4x+2$ $(2x+2)^2 = 4x^2+8x+4$ Square first then distribute!
$(2x+2)^2 \neq 2(x+1)^2$	See the previous example. You can not factor out a constant if there is a power on the parenthesis!
$\sqrt{-x^2+a^2} \neq -\sqrt{x^2+a^2}$	$\sqrt{-x^2+a^2} = (-x^2+a^2)^{\frac{1}{2}}$ Now see the previous error.
$\frac{a}{\left(\frac{b}{c}\right)} \neq \frac{ab}{c}$	$\frac{a}{\left(\frac{b}{c}\right)} = \frac{\left(\frac{a}{1}\right)}{\left(\frac{b}{c}\right)} = \left(\frac{a}{1}\right)\left(\frac{c}{b}\right) = \frac{ac}{b}$
$\frac{\left(\frac{a}{b}\right)}{c} \neq \frac{ac}{b}$	$\frac{\left(\frac{a}{b}\right)}{c} = \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{1}\right)} = \left(\frac{a}{b}\right)\left(\frac{1}{c}\right) = \frac{a}{bc}$